# Math 33A Worksheet 8

**Exercise 1.** True or false:

(a) If $A$ is orthogonal then it is invertible.

(b) If $A$ is symmetric it is invertible.

(c) Let $V$ be a subspace of $\mathbb{R}^n$ with orthonormal basis $\{u_1, \dots, u_m\}$, and let $\{v_1, \dots, v_{n-m}\}$ be an orthonormal basis for $V^\perp$. Then $\{u_1, \dots, u_m, v_1, \dots, v_{n-m}\}$ is an orthonormal basis for $\mathbb{R}^n$.

(d) The entries of an orthogonal matrix are all less than or equal to 1 in absolute value.

(e) Let $V$ be a subspace of $\mathbb{R}^n$ and $B$ the matrix for orthogonal projection onto $V$. Then $B^2 = B$.

(f) Let $\mathscr{B} = \{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \right\}$ be an ordered basis for $\mathbb{R}^3$. Then $\begin{bmatrix} 1 \\ -8 \\ 3 \end{bmatrix}_{\mathscr{B}} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.

(g) If $v_1, \dots, v_m$ is a basis of unit length vectors for a subspace $V$, there is an orthonormal basis of $V$ containing the vectors $v_1$ and $v_2$.

(h) For all $v, w \in \mathbb{R}^n$, $\langle v, w \rangle^2 \leq ||v||^2 ||w||^2$ with equality if and only if $v, w$ are perpendicular.

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**Exercise 2.** Let

$$A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 2 & 1 & 1 & -4 \end{bmatrix}$$

(a) Find an orthonormal basis $\mathscr{B} = \{u_1, u_2\}$ for $\ker A$.

(b) Using your basis from part (a), find the matrix $B$ for orthogonal projection onto $\ker A$.

(c) Find $B_{\mathscr{B}}$.

(d) **(Challenge).** Generalize your observation from part (c). Given an orthonormal basis $\mathscr{B} = \{u_1, \ldots, u_m\}$ for a subspace $V$, what is the matrix for orthogonal projection onto $V$ in the basis $\mathscr{B}$?

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**Exercise 3.** Find the QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

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**Exercise 4.** Let $A = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 0 & -1 & 2 & 1 \\ 1 & 4 & -6 & -3 \end{bmatrix}$, $V = \operatorname{im} A$.

(a) Find the projection matrix $B$ for $\operatorname{proj}_W$, projection onto $V$.

(b) Using $B$, determine whether $\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \in \operatorname{im} A$ (since $B$ is projection onto $V$, a vector $v$ is in $V$ if and only if $B \cdot v = v$).

(c) Find the least squares solution to $Ax = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$.

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